[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5042-A **D** **Your Roll No...............**

Unique Paper Code : 235466

Name of the Course : **B.Sc. (H) Comp. Sc., B.Sc. (Appl. Phy. Sc.) Analytical Chemistry/Industrial Chemistry/B.Sc. Mathematical Science/B.Sc. Physical Science**

Name of the Paper : MAPT-404 : Differential Equations

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **two** parts from each question.
3. **All** questions are compulsory.
4. Marks are indicated against each question.

UNIT – I

1. (a) Solve the initial value problem

$$(ye^x + 2e^x + y^2)dx + (e^x + 2xy)dy = 0,\ y(0) = 6.$$ (6½)

(b) Solve

$$y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$$ (6½)

(c) Solve

$$y = (1 + p)x + p^2.$$ (6½)

P.T.O.

2. (a) Solve

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x.$$

(6½)

(b) Solve :

$$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 4\ln x.$$

(6½)

(c) Consider the differential equation :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0,$$

(i) Show that e^x and xe^x are linearly independent solutions of this equation over R.

(ii) Write the general solution of the given equation,

(iii) Find the solution that satisfy the conditions $y(0) = 1$, $y'(0) = 4$. Explain why this solution is unique ?

(6½)

3. (a) Using method of variation of parameters, solve the differential equation

$$\frac{d^2y}{dx^2} + y = \tan^2 x.$$

(6½)

(b) Given that $y = e^{2x}$ is a solution of

$$(2x+1)\frac{d^2y}{dx^2} - 4(x+1)\frac{dy}{dx} + 4y = 0,$$

find a linearly independent solution by reducing the order. Write the general solution.

(6½)

(c) An 8- lb weight is placed upon the lower end of a coil spring suspended from the ceiling. The weight comes to the rest in its equilibrium position, there by stretching the spring 6 inch. The weight is then pulled down 3 inch below its equilibrium position and released at $t = 0$ with an initial velocity of 1 ft /sec. Neglecting the resistance of the medium and assuming that no external forces are present, determine the amplitude, period and frequency of the resulting motion. (6½)

4. (a) Solve

$$\frac{dx}{dt}+\frac{dy}{dt}+2x+y=0$$ (6½)

$$\frac{dy}{dt}+5x+3y=0$$

(b) Solve

$$(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2z\,dz = 0.$$ (6½)

(c) Solve

$$\frac{a\,dx}{(b-c)yz}=\frac{b\,dy}{(c-a)zx}=\frac{c\,dz}{(a-b)xy}.$$ (6½)

UNIT – II

5. (a) Find the partial differential equation of the system of right circular cones whose axes coincide with the line Oz. (5½)

(b) Find the general solution of the differential equation

$$(y + zx)p - (x + yz)q = x^2 - y^2.$$ (5½)

(c) Find the complete integral of the equation

$$p^2q(x^2 + y^2) = p^2 + q.$$ (5½)

P.T.O.

6. (a) Find the complete integral of the equation

$$2(z + xp + yq) = yp^2.$$ (6)

(b) Show that the equations

$$z = xp + yq, \qquad 2xy(p^2 + q^2) = z(xp + yq)$$

are compatible and find their solution. (6)

(c) Reduce the equation

$$\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$$

to canonical form. (6)